EXPONENTIAL SERIES

EXPONENTIAL & LOGARITHMIC SERIES

1. The number e

The sum of the infinite series $1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+....+\infty$ is denoted by the number e.

i.e., $e=\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^n=1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+....+\infty$

$$\therefore \quad e=\sum_{n=0}^{\infty}\frac{1}{n!}=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

Note :

(i) The number lies between 2 and 3. Approximate value of e = 2.718281828.

(ii) e is an irrational number. (i.e., $e\notin Q$)

2. Exponential Series

Expansion of any power x to the number e is the exponential series.

i.e., $e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+....\infty=\sum_{n=0}^{\infty}\frac{x^n}{n!}$ (where $x\in R$)

(i) Exponential theorem :

Let a > 0 then for all real value of x,

$$a^x=1+x(\log_e a)+\frac{x^2}{2!}(\log_e a)^2+\frac{x^3}{3!}(\log_e a)^3+....=\sum_{n=0}^{\infty}\frac{(x\log_e a)^n}{n!}$$

(ii) Some standard deductions from exponential series :

(i) $e^{-x}=1-\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+....+\frac{(-1)^n}{n!}x^n+....\infty$ {Replace x by – x)

$$e^{-x}=\sum_{n=0}^{\infty}(-1)^n.\frac{x^n}{n!}$$

(ii) $e=1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+....\infty$ {Putting x = 1 in (e^x)}

$$e=\sum_{n=0}^{\infty}\frac{1}{n!}$$

(iii) $e^{-1}=1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+....\infty$ {Putting x = –1 in (e^x)}

$$e^{-1}=\sum_{n=0}^{\infty}(-1)^n.\frac{1}{n!}$$ x = –1

(iv) $\frac{e^x+e^x}{2}=1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+....\infty$

$$\frac{e^x + e^{-x}}{2} = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$$

(v) $$\frac{e + e^{-1}}{2} = 1 + \frac{1}{2!} + \frac{1}{4!} + \ldots\infty$$

$$\frac{e^1 + e^{-1}}{2} = \sum_{n=0}^{\infty} \frac{1}{(2n)!}$$

3. Logarithmic Series : If ($|x| < 1$), then

$$\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\infty = \sum_{n-1}^{\infty} (-1)^{n-1} . \frac{x^n}{n}$$ is called as logarithmic series.

Some standard deductions from logarithmic series :

(i) $$\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + \ldots\infty \qquad = -\sum_{n=1}^{\infty} \frac{x^n}{n}$$

(ii) $$\log(1+x) - \log(1-x) = \log\left(\frac{1+x}{1-x}\right) = \left(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\right)$$

(iii) $$\log(1+x) + \log(1-x) = \log(1-x^2) = -2\left(\frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \ldots\right)$$

Note :

(i) Naperian or Natural log can be converted into common by using following relation :

$$\log_{10} N = \log_e N \times 0.43429448$$

(ii) $$\log 2 = 1 - \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \ldots = \frac{1}{2.1} + \frac{1}{3.4} + \frac{1}{5.6} + \ldots .$$

MATHEMATICAL INDUCTION

Mathematical statement:

Statements invovling mathematical reations are known as the mathematical statements. For example: 2 divides 16, (x + 1) is a factor of $x^2 - 3x + 2$.

The priciple of mathematical induction:

(i) First principle of mathematical induction:-

Let P(x) be a statement involving the natural number in such that

(i) P(1) is true i.e. P(n) is true for n = 1.

(ii) P(m+1) is true whenever P(m) is true.

then P(n) is true for all natural numbers n.

(ii) Second principle of mathematical induction:

Let P(n) be a statement involving the natural number n such that

(i) P(1) is true

(ii) P(m+1) is true, whenever P(n) is true $\forall n$, where $1 \le n \le m$.

then P(n) is true for all natural numbers.